Feedback Controller for Damping Oscillations in Arrays of Coupled Neurons

E. Adomaitienė, G. Mykolaitis, S. Bumelienė, A. Tamaševičius

Department of Electronics, Center for Physical Sciences and Technology, Lithuania

E-mail: elena.tamaseviciute@ftmc.lt

Introduction. Synchronization of coupled oscillators is a common observation in a variety of fields in nature, science and engineering [1]. The phenomenon has been widely investigated in physical, electronic, chemical, and biological systems, where it has been found to occasionally can give rise to rather surprising effects. For example, too strong synchronization of neurons in the brain may end up in Parkinson's disease symptoms. The standard therapy for patients is electrical deep brain stimulation (DBS) with strong relatively high frequency (about 100 Hz) pulse trains. Unfortunately, this treatment is often accompanied with side effects. A large number of more sophisticated feedback and non-feedback techniques to avoid synchronization of interacting oscillators in general, and more specifically with the possible application to neuronal arrays, have been described in literature, e.g. [2 - 9].

In this paper, we describe an alternative method and its electronic implementation to damp the spiking FitzHugh–Nagumo (FHN) type neurons, more specifically to stabilize their unstable steady states.

Circuits. The general set-up for damping oscillations in a neuronal array is sketched in Fig. 1. Here we consider a simple array, composed of only three oscillators. However, the analysis can be easily extended to larger arrays as well. In Fig. 1 CN is a coupling node, in general, not accessible from outside directly, but via a resistance R_g. The individual oscillators are mean-field coupled to the CN via effective resistive elements R*, not shown in the diagram for simplicity. DN is an accessible damping node.

The diagram of the feedback controller FC is shown in Fig. 2. We emphasize that the controller is an essentially two-terminal device.

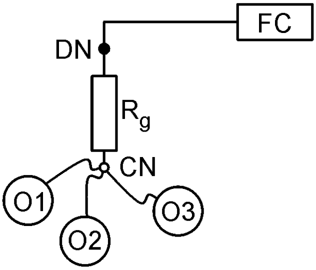

Fig. 1. Block diagram of three coupled oscillators O1, O2, and O3 with a feedback controller FC applied to the array

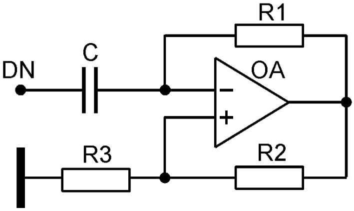

Fig. 2. Circuit diagram of the feedback controller FC. OA is a general-purpose operational amplifier, e.g. NE5534 type device. Capacitor C is set to ensure $R^*C >> T/2\pi$ (here T is the interspike period). $R_1 = R_2 = 1\ \text{k}\Omega$, resistor R_3 is adjusted to fit approximately the R_g.

Equations and numerical results. An individual FHN type asymmetric oscillator [10] is described by

$$\begin{aligned}\dot{x} &= ax - f(x) - y - c,\\ \dot{y} &= x - by.\end{aligned} \tag{1}$$

Here a, b and c are constant parameters. The common FHN activation function $F(x) = x - x^3$ is replaced with an asymmetric piecewise linear function $F(x) = ax - f(x)$ [10], where

$$f(x) = \begin{cases} d_1(x+1) & , \quad x < -1, \\ 0 & , \quad -1 \le x \le 1, \\ d_2(x-1) & , \quad x > 1. \end{cases} \tag{2}$$

Here the slope parameters are essentially different, $d_1 >> d_2$.

An array of N mean-field coupled FHN oscillators is given by

$$\begin{aligned}\dot{x}_i &= ax_i - f(x_i) - y_i - c_i + k_i(< x_i > - x_i),\\ \dot{y}_i &= x_i - by_i, \quad i = 1,..N.\end{aligned} \tag{3}$$

Here $<x_i>$ is the mean value of the variables x_i, the k_i are the coupling coefficients, further for simplicity assumed to be all equal $k_i = k$. The oscillators are supposed to be non-identical units due to different dc bias parameters c_i.

In the case the controller is applied to the array (Fig. 1), the equations read

$$\begin{aligned}\dot{x}_i &= ax_i - f(x_i) - y_i - c_i + k(z - x_i),\\ \dot{y}_i &= x_i - by_i, \quad i = 1,..N,\\ \dot{z} &= \omega_f k\left(\sum_{i=1}^{N}(x_i - z)\right) = \omega_f k\left(\sum_{i=1}^{N} x_i - Nz\right).\end{aligned} \tag{4}$$

Here z is the dimensionless voltage across the capacitor C of the controller, ω_f is the cut-off frequency of the filter, composed of R* and C. In the case the controller is switched off (formally, $C = 0$), the $\omega_f \rightarrow \infty$ and consequently

$$z = \frac{1}{N}\sum_{i=1}^{N} x_i \equiv < x_i > \qquad , \tag{5}$$

i.e. simply the mean value of the individual variables x_i, as expected.

Numerical simulation of Eq. (3) and Eq. (4) has been performed using the WOLFRAM MATHEMATICA 9 software. The results are presented in Fig. 3.

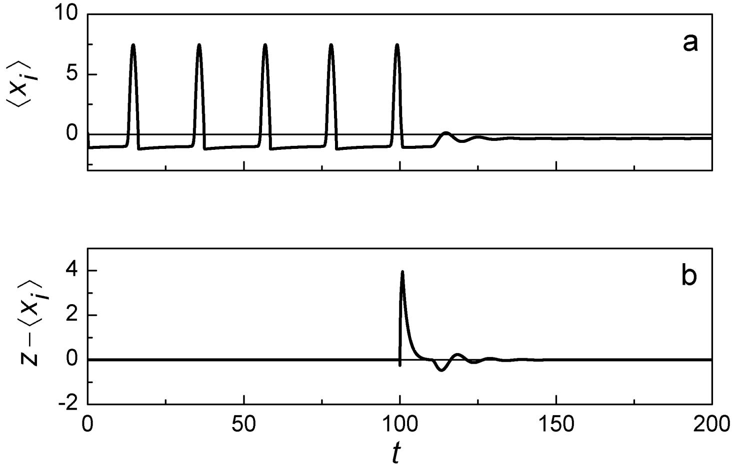

Fig. 3. Stabilizing unstable steady state in the array of coupled FHN oscillators. $N = 3$. $a = 4$, $b = 0.1$, $c_1 = 3.4$, $c_2 = 3.2$, $c_3 = 3.0$, $d_1 = 70$, $d_2 = 4$, $k = 5$, $\omega_f = 0.04$. (a) mean value $< x_i >$ of the variables x_i. (b) control signal, $z - < x_i >$. Controller is switched on at $t = 100$.

Concluding remarks. Concerning practical application of the described controller to real neuronal systems, we point out that the same electrode setup, as in the conventional DBS for the Parkinson's disease treatment [11, 12], can be used. The electrodes implanted in either *globus pallidus* or *subthalamic nucleus* of the brain can be readily exploited. The pulse generator, used for the DBS, should be replaced with the feedback circuit FC. An important advantage of the proposed technique over the DBS is that the control signals, sent into the brain from the FC, are vanishing.

In the recent papers [13, 14] detailed investigations, both analytical and numerical, have been carried out towards understanding the mechanism of the existing DBS therapy technique. We hope, that search for alternative methods can contribute to the problem of the treatment of the Parkinson's disease.

Future work. The next step is to perform both, SPICE based circuit design and experimental research of hardware electrical circuits, imitating FHN asymmetric oscillators and practically implemented feedback controller FC, especially using low-power consuming operational amplifiers. In addition, investigation of controlling larger neuronal arrays [15] is also planned in the nearest future.

Acknowledgement. This work was partly funded by a grant No. MIP–064/2013 from the Research Council of Lithuania.

References

1. Pikovsky A., Rosenblum M., Kurths J. Synchronization: A Universal Concept in Nonlinear Sciences. – Cambridge University Press, Cambridge, 2003.
2. Rosenblum M.G., Pikovsky A.S. Controlling synchronization in an ensemble of globally coupled oscillators // Phys. Rev. Lett., 2004. Vol. 92. – 114102.
3. Popovych O.V., Hauptmann C., Tass P. Effective desynchronization by nonlinear delayed feedback // Phys. Rev. Lett., 2005. Vol. 94. – 164102.
4. Tsimring L.S., Rulkov N.F., Larsen M.L. Repulsive synchronization in an array of phase oscillators // Phys. Rev. Lett., 2005. Vol. 95. – 014101.
5. Popovych O.V., Hauptmann C., Tass P.A. Control of neuronal synchrony by nonlinear delayed feedback // Biol. Cyber., 2006. Vol. 95. – P. 69–85.
6. Tass P.A. Phase Resetting in Medicine and Biology: Stochastic Modelling and Data Analysis. – Springer, Berlin, 2007.
7. Tukhlina N., Rosenblum M., Pikovsky A., Kurths J. Feedback suppression of neural synchrony by vanishing stimulation // Phys. Rev. E., 2007. Vol. 75. – 011918.
8. Pyragas K., Popovych O.V., Tass P.A. Controlling synchrony in oscillatory arrays with a separate stimulation – registration setup // Eur. Phys. Lett., 2007. Vol. 80. – 40002.
9. Tamaševičius A., Tamaševičiūtė E., Mykolaitis G. Feedback controller for destroying synchrony in an array of the FitzHugh–Nagumo oscillators // Appl. Phys. Lett., 2012. Vol. 101, – 223703.
10. Tamaševičius A., Tamaševičiūtė E., Mykolaitis G., Bumelienė S., Kirvaitis R., Stoop R. Neural spike suppression by adaptive control of an unknown steady state // Lecture Notes Comp. Sci., 2009. Vol. 5768. – P. 618–627.
11. Breit S., Schulz J.B., Benabid A.-L. Deep brain stimulation // Cell Tissue Res., 2004. Vol. 318. – P. 275–288.
12. Perlmutter J.S., Mink J.W. Deep brain stimulation // Annu. Rev. Neurosci., 2006. Vol. 29. – P. 229–257.
13. Ratas I., Pyragas K. Effect of high-frequency stimulation on nerve pulse propagation in the FitzHugh–Nagumo model // Nonlin. Dyn., 2012. Vol. 67. – P. 2899–2908.
14. Pyragas K., Novičenko V., Tass P. Mechanism of suppression of sustained neuronal spiking under high-frequency stimulation // Biol. Cyber., 2013. Vol. 107. – P. 669–684.
15. Tamaševičiūtė E., Mykolaitis G., Tamaševičius A. Analogue modelling an array of the FitzHugh–Nagumo oscillators // Nonlinear Analysis: Modelling and Control, 2012. Vol. 17, No.1. – P. 118–125.

Feedback Controller for Damping Oscillations in Arrays of Coupled Neurons

E. Adomaitienė, G. Mykolaitis, S. Bumelienė, A. Tamaševičius
Department of Electronics, Center for Physical Sciences and Technology, Lithuania

An extremely simple analog technique for stabilizing unstable steady states in an array of neuronal FitzHugh–Nagumo (FHN) type oscillators is described. A two-terminal feedback controller has been developed. The feedback circuit, when coupled to an array of oscillators, damps the spiking neurons, thus does away with the effect of synchronization. Numerical simulations have been performed. The results for an array of three mean-field coupled FHN type oscillators are presented.