Exploration of Modelling of Blood Flow through the Arterial Tree Using Phase Error Reducing Finite Element Model

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Introduction. It is known that modelling of cardiovascular system (for example, pulse wave propagation in blood vessels) can facilitate investigation of human physiology and help in diagnosis of various diseases [1].

This work explores the possibility to use a new phase error reducing variant of finite element model [2] to model the blood flow.

Materials and Methods. In this study blood flow is described using a one-dimensional fluid flow model considering the average flow velocity in the cross-section [2]. The compressibility of blood is estimated by equivalent fluid bulk modulus that takes into account not just compressibility of blood, but also elasticity of blood vessel walls:

$$\tilde{K} = K\left(1 + \frac{KD}{hE}\right)^{-1}, \tag{1}$$

where K is Young’s modulus of blood, D – diameter of the blood vessel, E – Young’s modulus of blood vessel wall, h– width of blood vessel wall.

The pressures and velocities of stabilised flow are found from the differential equation:

$$\left(\frac{1}{\rho} - \frac{v^2}{\tilde{K}}\right)\frac{\partial p}{\partial x} + \frac{f}{D}\frac{v|v|}{2} + g\sin a = 0, \tag{2}$$

where v is velocity of the flow, a – angle which blood vessel makes with a horizontal line, g – the free fall acceleration, $f = 0.3614/RE$ – friction, where $f = \rho D|v|/\mu$ is Reynold’s number (μ –dynamic viscosity of blood, ρ – density of blood).

Transient solution can be found using system of equations:

$$\begin{cases} \dfrac{\partial^2 p}{\partial t^2} + \dfrac{f}{D}|v|\dfrac{\partial p}{\partial t} - \dfrac{K}{\rho_0}\dfrac{\partial^2 p}{\partial x^2} = 0 \\ \dfrac{\partial v}{\partial t} = -\dfrac{1}{\rho_0}\dfrac{\partial p}{\partial x} - \dfrac{f}{D}\dfrac{v|v|}{2} - g\sin a. \end{cases} \tag{3}$$

This system of equations describes the case when pressure wave moves much faster than the flow velocity.

In order to create the finite element model that could perform the simulation of waves, at first the steady flow finite element model is to be created. This model finds the steady flow in accordance to the given boundary conditions (pressures or debits). In case of the steady flow the pressure and flow velocity at each point is constant and time-independent. The stiffness matrix of the element in that case is

$$\left[K_T^e\right]=\sqrt{\frac{D}{2fL|C|}}\left(A^2\rho-\frac{w_e^2}{\widetilde{K}}\right)\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}. \tag{4}$$

In order to find the steady flow results, the debit values have to be renewed in each Newton-Raphson iteration: $w_e^{(n)}=w_i^{(n-1)}$ (where n is number of iteration). It is also notable that the initial conditions of the steady flow, while calculated from the second equation of (3), also has to meet the conditions in the first equation. That can result in numerical errors. The simplest way to avoid them is to perform the transient simulation without additional stimulus. Damping will cause the model to reach a steady state when debits and pressures are consistent. Such steady state can be used as an initial condition for transient simulation with stimulus, that can be described by

$$\begin{cases}\left[M^e\right]\left\{\ddot{P}^e\right\}+\left[C^e\left(\dot{P}^e,P^e\right)\right]\left\{\dot{P}^e\right\}+\left[K^e\left(P^e\right)\right]\left\{P^e\right\}+\left\{Q^e\right\}=0\\ \dot{v}_e=G,\end{cases} \tag{5}$$

where $\{P^e\}$ is the pressure vector, $\{Q^e\}$ – external force vector. Other matrices and vectors from (5) can be found like this:

$$\left[M^e\right]=\frac{AL}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{6.1}$$

$$\left[C^e\right]=\frac{ALf|v_e|}{2D}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{6.2}$$

$$\left[K^e\right]=\frac{A\widetilde{K}}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}, \tag{6.3}$$

$$\left\{Q^e\right\}=A\widetilde{K}\left(\frac{p}{L}-\frac{fv_e|v_e|}{2D}-g\sin a\right)\begin{Bmatrix}1\\ -1\end{Bmatrix}, \tag{6.4}$$

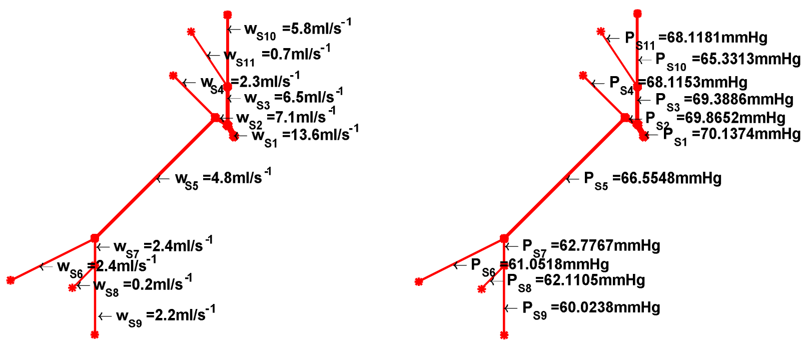
$$G^e=\frac{v_e}{\widetilde{K}}\frac{\dot{p}_1+\dot{p}_2}{2}-\frac{p_2-p_1}{\rho_0 L}-\frac{f}{D}\frac{fv_e|v_e|}{2}-g\sin a^2, \tag{6.5}$$

where p^e is the pressure stimulus in the element, p_1, p_2, $\dot{p}_1$, $\dot{p}_2$ – pressures and their derivatives in the nodes of the element, v_e – the flow velocity in the element. The first equation of system (5) is transformed into the equation describing the whole system, while the second one is solved for each element separately. It should be noted that (5) system does not include any algebraic relation that would keep the debit balance between neighbouring elements. However, for almost uncompressible fluids the distortion is minimal and it can be ignored. If it was more significant, corrections at each time step should be applied to ensure the nodal flow mass rate balance.

The wave non-reflection condition in the points where the arterial tree has been truncated can be evaluated like this:

$$\sqrt{\frac{\widetilde{K}}{\rho}}\frac{\partial p}{\partial x}+\frac{\partial p}{\partial t}=0\,. \tag{7}$$

The numerical experiment is performed using the model of a fragment of arterial tree (right shoulder and arm). The parameters of the model have been chosen taking into account the physical data of the subject (age – 25 years, height – 1.87 m, weight – 76 kg) and generic data given in literature [3, 4].The fragment being analysed consists of 11 blood vessels: Brachiocephalic (S1), R. subclavian (S2), R. common carotid (S3), R. vertebral (S4), R. brachial (S5), R. radial (S6), R. ulnar A (S7), R. interosseous (S8), R. ulnar B (S9), R. internal carotid (S10) and R. external carotid (S11) (since the model does not take much of their geometry into account, they can be represented by edges of a graph). The boundary conditions are given as the pressure for the beginning of S1 (p_{S1} = 70.25 mmHg) and debits for the ends of S4, S6, S8, S9, S10,and S11 (w_{S4} = 6.5 ml/s, w_{S6} = 2.4 ml/s, w_{S8} = 0.2 ml/s, w_{S9} = 2.2 ml/s, w_{S1} = 5.8 ml/s).The steady flow analysis using the given conditions results in the debits and pressures that are shown in Fig. 1.



(a) (b)

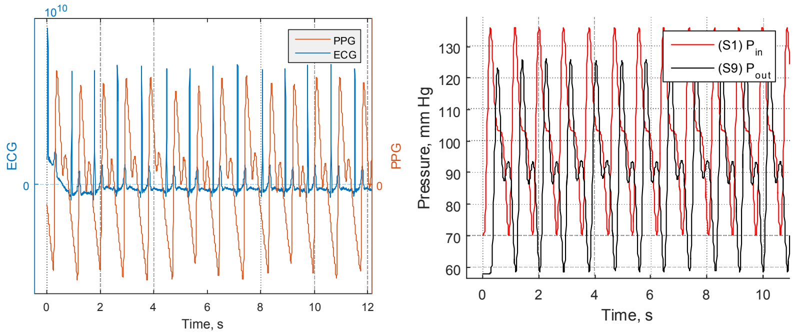
Fig. 1. Fragment of arterial tree with debits of the flow (a) and pressures(b)

The pressure stimulus was simulated using an analytical function:

$$u(t) = 1 + \sin\left(T2\pi t - \frac{4\pi}{15} + \sin\left(T2\pi t - \frac{4\pi}{15}\right)\right). \tag{8}$$

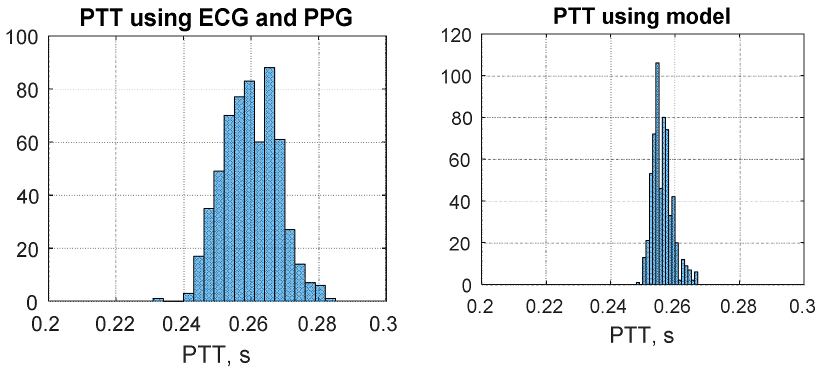
During the transient simulation the beginning of S1 receives a periodical stimulus described by (8) where the time T is modified according to the ECG signal (so that T waves would correspond to the rising slope) and the ends of S10, S1, S4, S6, S8, S9 are assigned the wave non-reflection condition.

Results. Fig 2a shows the synchronised electrocardiogram (ECG) and photoplethysmogram (PPG) signals (PPG has been taken on a finger). Fig. 2b shows the resulting pressures in the beginning of blood vessel S1 (that is, the stimulus) and end of blood vessel S9 (corresponding to the finger).



(a) (b)

Fig. 2. ECG and PPG signals (a) and results of simulation (b)



(a) (b)

Fig. 3. Histograms of PTT found using the ECG and PPG (a) and PTT found using the Finite element model(b)

Analysis of those results can find the distances between local maximums of pressure in both points (Pulse transit time, PTT).

Fig. 3 compares the PTT histograms of the sample of 599 R-R intervals.

It can be seen that the PTT values are rather similar (root mean square error between those intervals is 0.0037 s). That makes it possible to reach a conclusion that the method is likely to be suitable for modelling of the arterial tree.

Conclusions. The possibility to use a new phase error reducing variant of finite element model to model the blood flow has been explored. It has been found that, even using mostly generic anatomical data, the method can be adapted to find the values of pulse transit time that are close to the ones found from ECG and PPG.

The future work should include finding a good way to take into account more personalised anatomical data and comparison of the proposed method with alternatives.

References

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